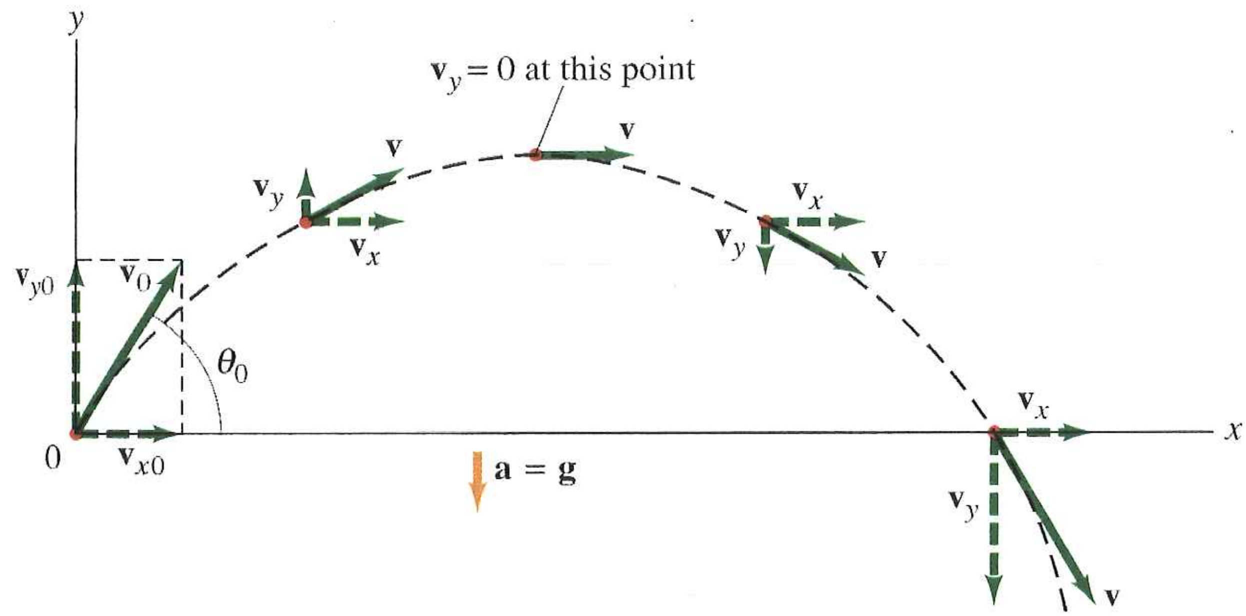
Honors Physics

Ch 3. Part II: Vector Applications

Lesson 1: Projectile Motion

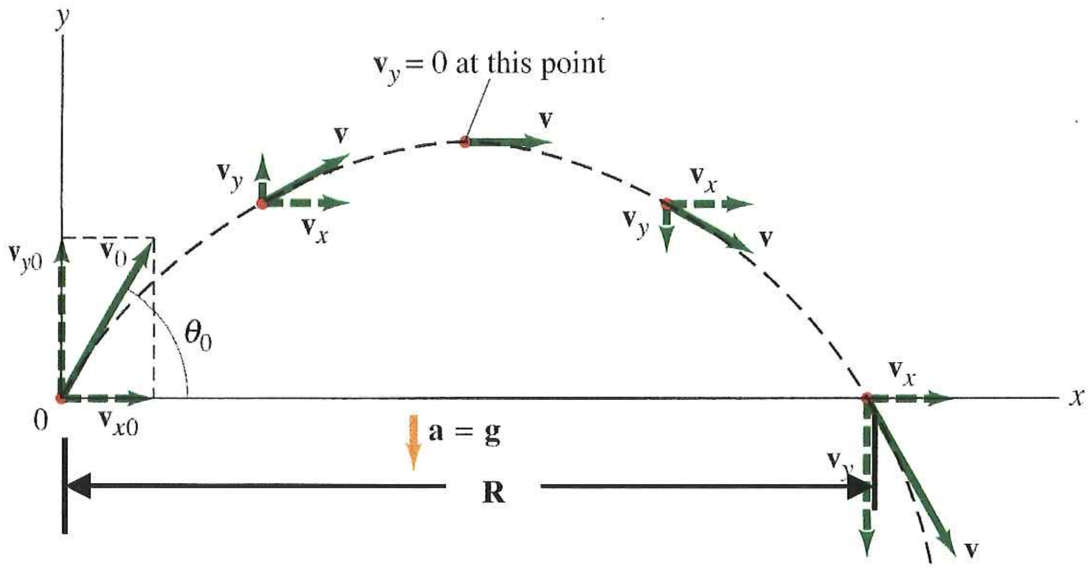


Problem: A particle is given an initial velocity $\boldsymbol{v_o}$ at some angle, θ_0, from the horizontal.

Assumptions: 1. Particle is a point particle (No rotation or vibration).

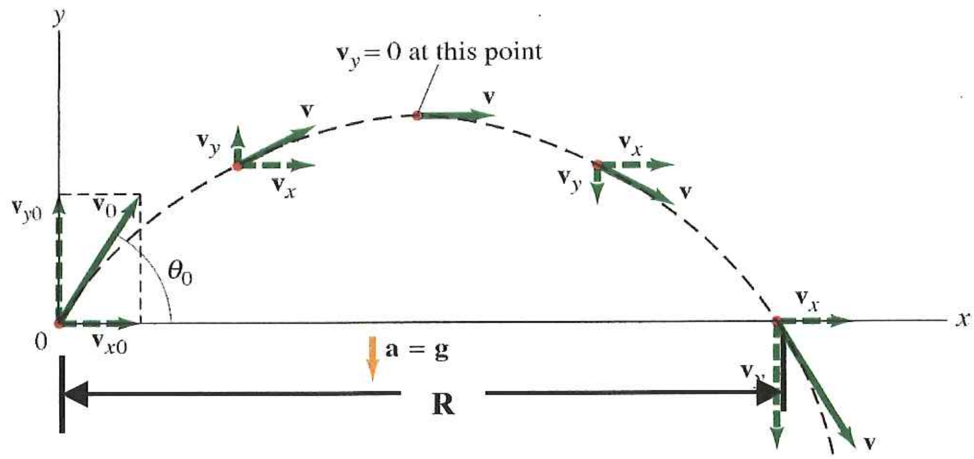
2. Neglect air resistance and wind.

3. Acceleration due to gravity is constant, $-9.8\frac{m}{s^2}\hat{y}$



X-Direction	Y-Direction
$a_x = 0$	$a_x = -\ 9.8\ \ m/s^2\ \ \hat{y}$
$v_{fx} = v_{ix} + a_x t$	$v_{fy} = v_{iy} + (-9.8)t$
$x_f = x_i + v_{ix} t + \frac{1}{2} a_x t^2$	$y_f = y_i + v_{iy} t - \frac{1}{2}(9.8)t^2$
$\Delta x = v_{ix} t + \frac{1}{2} a_x t^2$	$\Delta y = v_{iy} t + \frac{1}{2}(-9.8)t^2$

PROJECTILE MOTION – Special Case: $\Delta y = 0$



IF $x_i = 0$, $y_i = 0$ and $y_f = 0$ then $x_f = R$ *(Range)*.

$$y_f = v_i(\sin\theta)t - \tfrac{1}{2}gt^2$$

$$0 = t\left[v_i(\sin\theta) - \tfrac{1}{2}gt\right]$$

$$v_i(\sin\theta) - \tfrac{1}{2}gt = 0 \quad \text{now solve for } t$$

$$\boxed{t = \frac{2v_i\sin\theta}{g}} \quad \Leftarrow \text{time for total Flight}$$

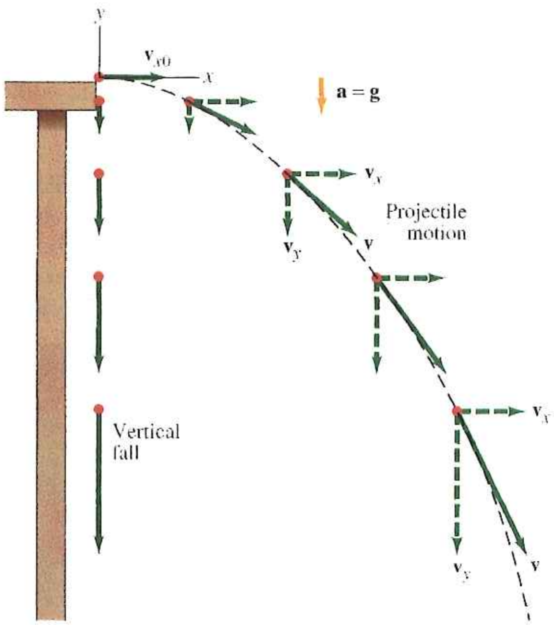
Subitute into $x_f = x_i + v_i(\cos\theta)t$ where $x_f = R$

$$R = v_i\cos\theta\frac{2v_i\sin\theta}{g} = \frac{v_i^2[2\cos\theta\sin\theta]}{g} \quad \text{trig. identity?}$$

Recall? $2\cos\theta\sin\theta = \sin 2\theta$

Finally $\Rightarrow$ $\boxed{R = \dfrac{v_i^2\sin 2\theta}{g}}$ $\Leftarrow$ the RANGE formula!

<u>CLASSIC BLUNDER…</u> If a bullet was fired horizontally from the shoulder of a tall man, and at the moment it left the barrel, a second identical bullet was dropped from the same height. Which bullet would reach the ground first?

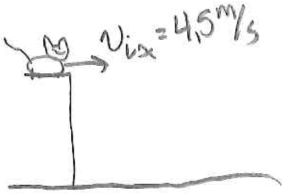


Solution:

- Gravity is the only force acting on both bullets!
- Both bullets have the same initial conditions in the y – plane.
- Therefore… <u>Same time</u> in the air!

Lesson 1 Problems: Projectile Motion

1. (I) A tiger leaps horizontally from a 7.5 m high rock with a speed of 4.5 m/s. How far from the base of the rock will she land?



HORIZONTAL

$v_{ix} = 4.5\,m/s$

$a_x = 0\,m/s^2$

$t = 1.237\,sec$

$\Delta x = ?$

$\Delta x = v_{ix} t + \frac{1}{2} a_x t^2$

$\Delta x = 4.5\,m/s\,(1.237\,s)$

$\boxed{\Delta x = 5.567\,m}$

VERTICLE

$\Delta y = -7.5\,m$

$a = -9.8\,m/s^2$

$v_{iy} = 0\,m/s$

FIND t: $\Delta y = v_{iy} t + \frac{1}{2}(-9.8)t^2$

$-7.5\,m = 0 + \frac{1}{2}(-9.8)t^2$

$t = 1.237\,sec$

2. (II) A ball is thrown horizontally from the roof of a building 56 m tall and lands 45 m from the base. What was the ball's initial speed?

HORIZONTAL

$\Delta x = 45\,m$

$a = 0\,m/s^2$

$t = 3.38\,sec$

$\Delta x = v_{ix} t + \frac{1}{2} a_x t^2$

$45 = v_{ix}(3.38\,s)$

$\boxed{v_{ix} = 13.311\,m/s}$

VERTICAL

$\Delta y = -56\,m$

$a = -9.8\,m/s^2$

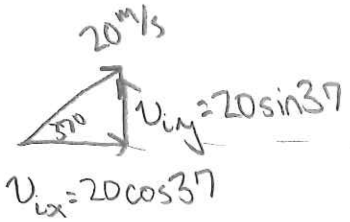
$v_{iy} = 0\,m/s$

FIND t: $\Delta y = v_{iy} t + \frac{1}{2} a t^2$

$-56\,m = 0(t) + \frac{1}{2}(-9.8)(t^2)$

$t = 3.38\,sec$

3. (II) A football is kicked at ground level with a speed of 20.0 m/s at and angle of 37.0° to the horizontal. How much later does it hit the ground?



HORIZONTAL

$v_{ix} = 15.973\ \text{m/s}$

$a_x = 0\ \text{m/s}^2$

VERTICAL

$v_{iy} = 12.036\ \text{m/s}$

$a = -9.8\ \text{m/s}^2$

$\Delta y = 0$

$v_{fy} = -12.036\ \text{m/s}$

$v_{fy} = v_{iy} + (-9.8)t$

$-12.036 = 12.036 - 9.8t$

$\boxed{t = 2.456\ \text{sec}}$

4. (II) A ball thrown horizontally at 22.2 m/s from the roof of a building lands 36.0 m from the base of the building. How high is the building?

HORIZONTAL

$v_{ix} = 22.2\ \text{m/s}$

$\Delta x = 36.0\ \text{m}$

$a = 0\ \text{m/s}^2$

$\Delta x = v_{ix}(t) + \frac{1}{2}a_x t^2$

$36 = 22.2t + \cancel{\frac{1}{2}(0)t^2}$

$t = 1.622\ \text{sec}$

VERTICAL

$v_{iy} = 0\ \text{m/s}$

$a = -9.8\ \text{m/s}^2$

$t = 1.622\ \text{sec}$

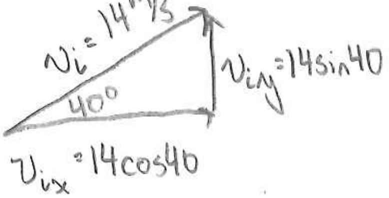
$\Delta y = v_{iy}t + \frac{1}{2}at^2$

$\Delta y = 0 + \frac{1}{2}(-9.8)(1.62)^2$

$\Delta y = -12.885\ \text{m}$

$\boxed{12.885\ \text{m TALL}}$

5. (II) A shot-putter throws the shot with an initial speed of 14 m/s at a 40° angle to the horizontal. Calculate the horizontal distance traveled by the shot if it leaves the athlete's hand at a height of 2.2 m above the ground.



HORIZONTAL

$v_{ix} = 10.72\ m/s$

$a_x = 0\ m/s^2$

$t = 2.055\ sec$

$\Delta x = ?$

$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2$

$\Delta x = 10.72(2.055) + 0$

$\boxed{\Delta x = 22.039\ m}$

VERTICAL

$v_{iy} = 8.999\ m/s$

$a_y = -9.8\ m/s^2$

$\Delta y = -2.2\ m$

$t = ?$

$\Delta y = v_{iy}t + \frac{1}{2}(-9.8)t^2$

$-2.2\ m = (8.999\ m/s)t + \frac{1}{2}(-9.8)t^2$

$0 = -4.9t^2 + 8.999t + 2.2$

QUAD FORM

$t = -.2185,\ 2.055$

6. (II) A hunter aims directly at a target (on the same level) 240 m away. (a) If the bullet leaves the gun at a speed of 250 m/s, by how much will it miss the target? (b) At what angle should the gun be aimed so the target will be hit?

a.

HORIZONTAL

$v_{ix} = 250\ m/s$

$a_x = 0\ m/s^2$

$\Delta x = 240\ m$

$\Delta x = v_{ix}t + \cancel{\frac{1}{2}at^2}$

$240\ m = (250\ m/s)t$

$t = .96\ sec$

VERTICAL

$v_{iy} = 0\ m/s$

$a_y = -9.8\ m/s^2$

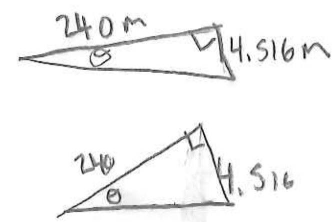
$t = .96\ sec$

$\Delta y = \cancel{v_{iy}t} + \frac{1}{2}at^2$

$\Delta y = \frac{1}{2}(-9.8)(.96)^2$

$\boxed{\Delta y = -4.516\ m}$

b.



b. $v_{ix} = 250\cos\theta$

$a_x = 0\ m/s^2$

$\Delta x = 240\ m$

$240\ m = 250\cos\theta\ t$

$t = .96\cos\theta$

$\Delta y = 0$

$v_{iy} = 250\sin\theta$

$a = -9.8\ m/s^2$

$t = .96\cos\theta$

$v_{fy} = v_{iy} + (-9.8)t$

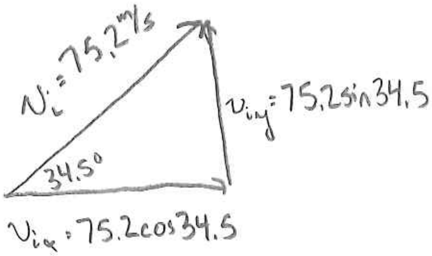
$-250\sin\theta = 250\sin\theta + (-9.8)(.96\cos\theta)$

$-500\sin\theta = -9.408\cos\theta$

$\tan\theta = .018816$

$\therefore \boxed{\theta = 1.078°}$

7. (II) A projectile is fired with an initial speed of 75.2 m/s at an angle of 34.5° above the horizontal on a long flat firing range. Determine (a) the maximum height reached by the projectile, (b) the total time in the air, (c) the total horizontal distance covered (that is, the range), and (d) the velocity of the projectile 1.50 s after firing.



HORIZONTAL

$v_{ix} = 61.974$ m/s

$a_x = 0$ m/s²

$t = 8.692$ sec

$\Delta x = v_{ix} t + \frac{1}{2} a_x t^2$

$\Delta x = 61.974(8.692)$

c. $\boxed{\Delta x = 538.695\text{m}}$

VERTICAL

$v_{iy} = 42.594$ m/s

$a_y = -9.8$ m/s²

$v_{fy} = 0$ m/s

FIND t: $v_{fy} = v_{iy} + at$

$0 = 42.594 + (-9.8)t$

$t = 4.346$ sec

SO $\Delta y = v_{iy} t + \frac{1}{2} a t^2$

$= 42.594(4.346) + \frac{1}{2}(-9.8)(4.346)^2$

$= 185.1135 \quad -92.564$

a. $\boxed{\Delta y = 92.55\text{m}}$

b. $t = 4.346\text{sec} * 2 \Rightarrow \boxed{t = 8.692\text{sec}}$

d. $t = 1.5$s

$v_{fx} = v_{ix} + a_x t$

$v_{fx} = 61.974 + 0(1.5)$

$v_{fx} = 61.974$ m/s

$t = 1.5$s

$v_{fy} = v_{iy} + a_y t$

$v_{fy} = 42.594 + (-9.8)(1.5)$

$v_{fy} = 27.894$ m/s

$v_f = \sqrt{61.974^2 + 27.894^2}$

$= \sqrt{3840.7766\ 778.0752} \Rightarrow \boxed{v_f = 67.96\text{ m/s}}$

8. (II) A projectile is shot from the edge of a cliff 125 m above the ground level with an initial speed of 105 m/s at and angle of 37.0° with the horizontal, as shown in Fig. 3-39. (a) Determine the time taken by the projectile to hit point P at ground level. (b) Determine the range X of the projectile as measured from the base of the cliff. At the instant just before the projectile hits point P, find (c) the horizontal and the vertical components of its velocity, (d) The magnitude of the velocity, and (e) the angle made by the velocity vector with the horizontal.

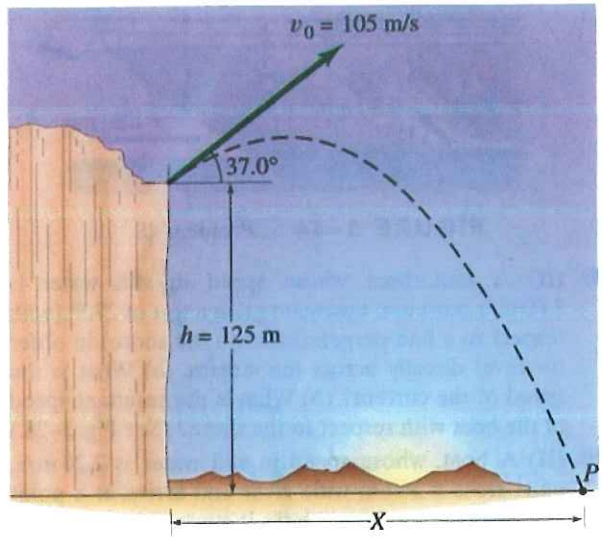
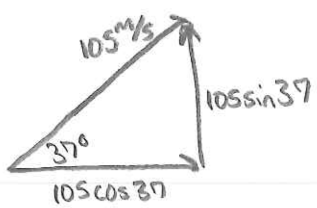


FIGURE 3-39



HORIZONTAL

$v_{ix} = 83.857 m/s$

$a_x = 0 m/s^2$

$t = 14.6333 sec.$

$\Delta x = v_{ix} t + \frac{1}{2} a_x t^2$

$\Delta x = (83.857)(14.6333)$

$\boxed{\Delta x = 1227.099 m}$

$v_{fx} = ?$

$v_{fx} = v_{ix} + a_x t$

$\boxed{v_{fx} = 83.857 m/s} + 0$

VERTICAL

$\Delta y = -125 m$

$v_{iy} = 63.1906 m/s$

$a_y = -9.8 m/s^2$

FIND t: $\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$

$-125 m = 63.1906 t + \frac{1}{2}(-9.8) t^2$

QUAD FORMULA

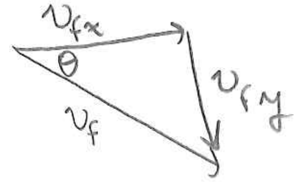
$t = -1.743, \boxed{14.6333 sec.}$

$v_{fy} = ?$

$v_{fy} = v_{iy} + a_y t$

$v_{fy} = 63.1906 + (-9.8)(14.6333)$

$\boxed{v_{fy} = -80.2155 m/s}$



$v_f = \sqrt{83.857^2 + 80.2155^2}$

$\therefore \boxed{v_f = 116.05 m/s}$

$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right)$

$\theta = \tan^{-1}\left(\frac{80.2155}{83.857}\right)$

$\therefore \boxed{\theta = 43.729°}$

Lesson 2: Relative Motion

Determine which vector the problem is asking for (What with respect to what?).

- Using the subscript method we can develop a vector diagram with what you are looking for as the resultant.
- Remember: $\vec{v}_{SB} = -\vec{v}_{BS}$
- Now treat the problem just like a vector component problem.

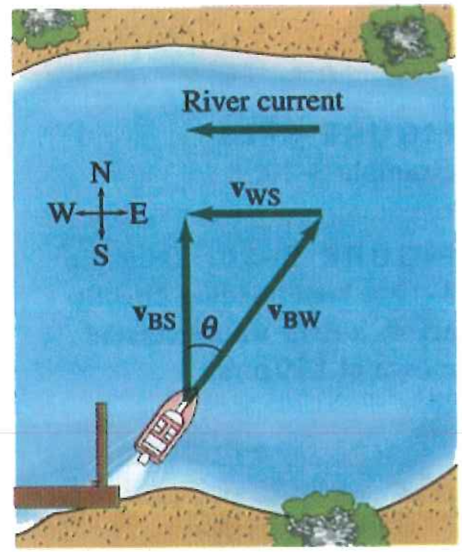
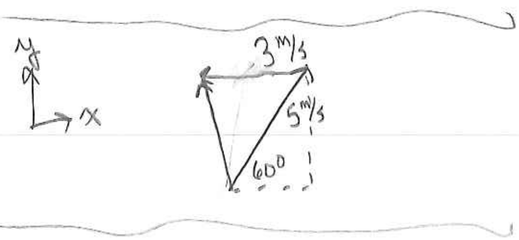


FIGURE 3–28 The boat must head upstream at an angle θ if it is to move directly across the river. Velocity vectors are shown as green arrows:
$\mathbf{v}_{BS}$ = velocity of **B**oat with respect to the **S**hore,
$\mathbf{v}_{BW}$ = velocity of **B**oat with respect to the **W**ater,
$\mathbf{v}_{WS}$ = velocity of the **W**ater with respect to the **S**hore (river current).

Example Problem – Relative Velocities:

A boat points itself upstream 60° from the shoreline in a river whose current is 3.0 m/s. The boat is capable of traveling at 5.0 m/s in still water. (a) What is the velocity of the boat with respect to the shore? (b) If the river is 86.7 m wide, how long will it take to cross the river? (c) How far down stream will the boat be from a point directly across from where it started?



$$v_{BW} = 5\cos 60\hat{x} + 5\sin 60\hat{y}$$

$$v_{BW} = 2.5\hat{x} + 4.33\hat{y}$$

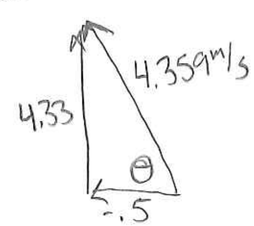
$$v_{WS} = -3\hat{x}$$

$$v_{BS} = -.5\hat{x} + 4.33\hat{y}$$

$$v_{BS} = \sqrt{.5^2 + 4.33^2} = \boxed{4.359\ m/s}$$

$$v = \frac{d}{t} \qquad 4.333\ m/s = \frac{86.7\ m}{t} \Rightarrow t = \boxed{20.0\ sec}$$

VELOCITY:

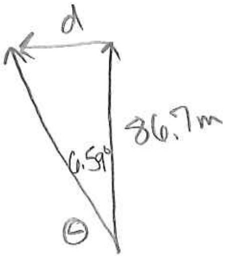


$$\theta = \tan^{-1}\left(\frac{4.33}{.5}\right) = 83.413°$$

$$\tan 6.587 = \frac{d}{86.7}$$

$$\boxed{d = 10.012\ m}$$

POSITION:



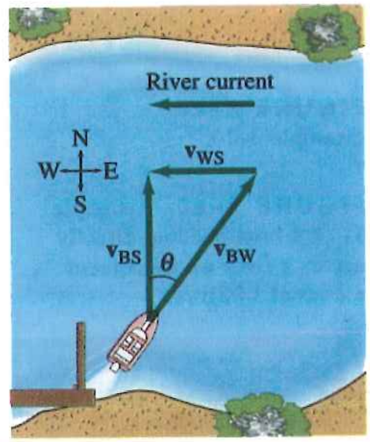
- OR -

$$\tan 83.413 = \frac{86.7}{d}$$

$$\boxed{d = 10.012\ m}$$

Lesson 2: Relative Motion

1. (II) A motorboat whose speed in still water is 3.60 m/s must aim upstream at an angle of 27.5° (with respect to a line perpendicular to the shore) in order to travel directly across the stream. (a) What is the speed of the current? (b) What is the resultant speed of the boat with respect to the shore? (See Fig. 3-28)



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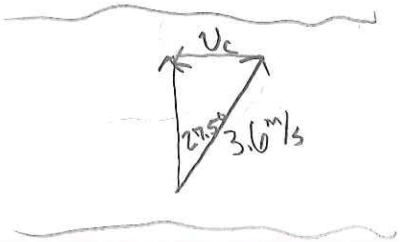


a. CURRENT: $\sin 27.5 = \dfrac{v_c}{3.6\,m/s}$

$$v_c = \boxed{1.662\ m/s}$$

b. $v_{BS} \Rightarrow \cos 27.5 = \dfrac{v_{BS}}{3.6}$

$$v_{BS} = \boxed{3.19\ m/s}$$

2. (II) Huck Finn walks at a speed of 1.0 m/s across his raft (that is, he walks perpendicular to the raft's motion relative to the shore). The raft is traveling down the Mississippi River at a speed of 2.7 m/s relative to the river back (Fig. 3-43). What is the velocity (speed and direction) of Huck relative to the river bank?

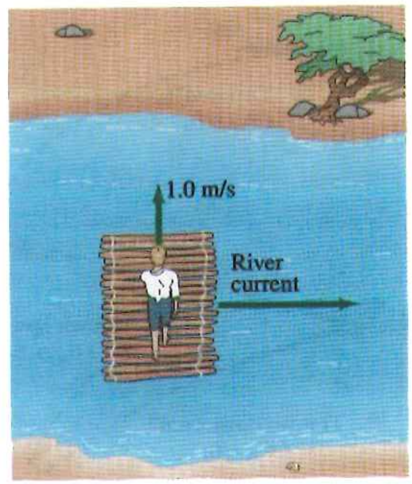
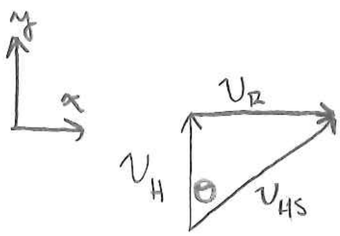


FIGURE 3–43



$$v_H = 0\,m/s\,\hat{x} + 1\,m/s\,\hat{y}$$

$$+\,v_R = 2.7\,m/s\,\hat{x} + 0\,m/s\,\hat{y}$$

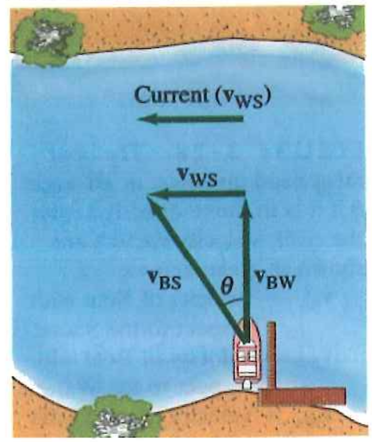
$$v_{HS} = 2.7\,m/s\,\hat{x} + 1\,m/s\,\hat{y}$$

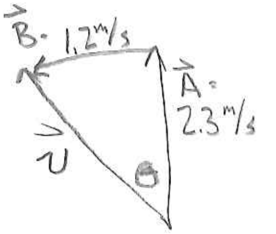
$$v_{HS} = \sqrt{2.7^2 + 1^2} = \boxed{2.879\,m/s}$$

$$\theta = \tan^{-1}\left(\frac{2.7}{1}\right) = \boxed{69.68^\circ}$$

3. (II) A boat can travel 2.30 m/s in still water. (a) If the boat points its prow directly across a stream whose current is 1.20 m/s, what is the velocity (magnitude and direction) of the boat relative to the shore? (b) What will be the position of the boat, relative to its point of origin, after 3.00 s? (See Fig. 3-30

FIGURE 3-30





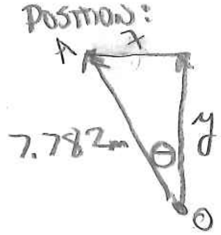
$$\vec{A} = 0\,\tfrac{m}{s}\,\hat{x} + 2.3\,\tfrac{m}{s}\,\hat{y}$$
$$+\ \vec{B} = -1.2\,\tfrac{m}{s}\,\hat{x} + 0\,\tfrac{m}{s}\,\hat{y}$$
$$\vec{v} = -1.2\,\tfrac{m}{s}\,\hat{x} + 2.3\,\tfrac{m}{s}\,\hat{y}$$

a. $v = \sqrt{1.2^2 + 2.3^2} = \boxed{2.59\,\tfrac{m}{s}}$

$\theta = \tan^{-1}\left(\frac{1.2}{2.3}\right) = \boxed{27.55^\circ}$

b. $v = \frac{d}{t} \Rightarrow 2.59 = \frac{d}{3} \Rightarrow d = 7.782\,m$

POSITION:



$x = 7.782 \sin 27.55 = \boxed{3.6\,m\,\hat{x}}$

$y = 7.782 \cos 27.55 = \boxed{6.9\,m\,\hat{y}}$

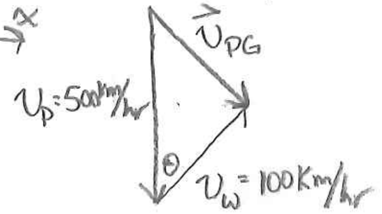
4. (II) Two planes approach each other head-on. Each has a speed of 835 km/h, and they spot each other when they are initially 10.0 km apart. How much time do the pilots have to take evasive action?

$\xrightarrow{835\,km/hr}$ $\xleftarrow{835\,km/hr}$ IS SAME AS $\xrightarrow{1670\,km/hr}$ 0 km/hr

SO $v = \frac{d}{t}$

$1670\,\tfrac{km}{hr} = \frac{10\,km}{t} \Rightarrow t = .0059\,hr = .36\,min = \boxed{22\,sec}$

5. (II) An airplane is heading due south at a speed of 500 km/h. If a wind begins blowing from the southwest at a speed of 100 km/h (average), calculate: (a) the velocity (magnitude and direction) of the plane relative to the ground, and (b) how far off course it will be after 10 min if the pilot takes no corrective action. [Hint: first draw a diagram].



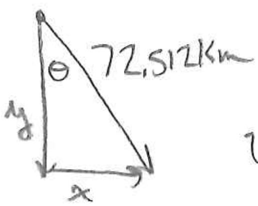
$$\vec{v}_p = 0\ Km/hr\ \hat{x} + -500\ Km/hr\ \hat{y}$$

$$\vec{v}_w = 100 \sin 45\ Km/hr\ \hat{x} + 100 \cos 45\ \frac{Km}{hr}\ \hat{y}$$

$$\vec{v}_{PG} = 70.7\ Km/hr\ \hat{x} + -429.289\ Km/hr\ \hat{y}$$

a. $v_{PG} = \sqrt{70.7^2 + 429.289^2} = \boxed{435.07\ Km/hr}$

$\theta = \tan^{-1}\left(\frac{70.7}{429.289}\right) = \boxed{9.35°}$

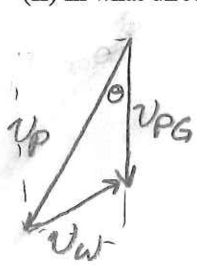


$v = \frac{d}{t} \rightarrow 435.07 = \frac{d}{.166} \rightarrow d = 72.512\ Km$

$x = 72.512\ Km \sin 9.35 = \boxed{11.78\ Km\ \hat{x}}$

$y = -72.512 \cos 9.35 = \boxed{-71.55\ Km\ \hat{y}}$

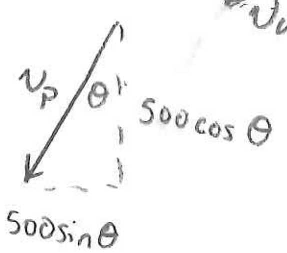
6. (II) In what direction should the pilot aim the plane in problem 5 so that it will fly due south?



$$v_p = 500 \sin\theta\ \hat{x} + 500 \cos\theta\ \hat{y}$$

$$+\ v_w = 100 \sin 45\ \hat{x} + 100 \cos 45\ \hat{y}$$

$$v_{PG} = 0\ \hat{x} + \_\_\_\_\ \hat{y}$$



So $500 \sin\theta + 100 \sin 45 = 0$

$500 \sin\theta = -70.71$

$\sin\theta = .1414$

$\therefore \boxed{\theta = 8.13°}$ INTO THE WIND (SW)